Prague Conference on Convex Geometry

Book of Abstracts and Program

August 24–28, 2026
Prague, Czech Republic

Basic Information

Conference Title: Prague Conference on Convex Geometry (PCCG)
Date: August 24–28, 2026
Location: Charles University, Malá Strana campus (lecture hall S9)
Malostranské nám. 2/25, Praha 1
Website: https://pccg.karlin.mff.cuni.cz

Main speakers

Károly Böröczky (Rényi Institute of Mathematics)

Andrea Colesanti (University of Florence)

Jonas Knoerr (TU Wien)

Jan Rataj (Charles University)

Christos Saroglou (University of Ioannina)

Ramon van Handel (Princeton University)

Peter van Hintum (ETH Zürich)

Dongmeng Xi (Shanghai University)

Organizers

Jan Kotrbatý (Charles University)

Thomas Wannerer (Friedrich-Schiller-Universität Jena)

List of Participants

Semyon Alesker
Gergely Ambrus
Rafik Aramyan
Silvana Aramyan
Egor Bakaev
Dominik Beck
Florian Besau
Károly Böröczky
Leo Brauner
Christian Buchta
Xiaxing Cai
Ruijia Cao
Jiaxuan Chen
Zhang Chen
Andrea Colesanti
Giacomo Di Paolo
Dmitry Faifman
Ferenc Fodor
Ansgar Freyer
Filip Fryš
Bernardo González Merino
Florian Grundbacher
Florian Grünstäudl
Agam Guberman
Georg Hofstätter
Lawrence Hollom
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Mohamed Mouamine
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Daniel Papvari
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Monica Fiorella Martinez Sanchez
Jorge Santiago Ibáñez Marcos
Christos Saroglou
Alon Schejter
Maud Szusterman
Kateryna Tatarko
Marius Tiba
Jacopo Ulivelli
Viktor Vígh
Ramon van Handel
Peter van Hintum
Katherina von Dichter
Shanshan Wang
Dongmeng Xi
Tim Žiaran

Program

Monday, August 24

Time	Speaker / Activity
08:30	Registration
09:00	**Károly Böröczky**
10:00	**Oscar Ortega**
10:30	Coffee
11:00	**Gergely Ambrus**
11:30	**Leo Brauner**
12:00	**Ferenc Fodor**
12:30	Lunch
14:30	**Andrea Colesanti**
15:30	**Ansgar Freyer**
16:00	Coffee
16:30	**Jacopo Ulivelli**
17:00	**Agam Guberman**
17:30	**Jorge Santiago Ibáñez Marcos**

Tuesday, August 25

Time	Speaker / Activity
09:00	**Jonas Knoerr**
10:00	**Marius Tiba**
10:30	Coffee
11:00	**Semyon Alesker**
11:30	**Florian Besau**
12:00	**Jin Li**
12:30	Lunch
14:30	**Christos Saroglou**
15:30	**Rafik Aramyan**
16:00	Coffee and Poster Session

Wednesday, August 26

Time	Speaker / Activity
09:00	**Ramon van Handel**
10:00	**Dmitry Faifman**
10:30	Coffee
11:00	**Florian Grundbacher**
11:30	**Bernardo González Merino**
12:00	**Katherina von Dichter**

Thursday, August 27

Time	Speaker / Activity
09:00	**Peter van Hintum**
10:00	**Christian Kipp**
10:30	Coffee
11:00	**Zsolt Langi**
11:30	**Mohamed Mouamine**
12:00	**Marton Naszodi**
12:30	Lunch
14:30	**Dongmeng Xi**
15:30	**Alon Schejter**
16:00	Coffee
16:30	**Luca Iffland**
17:00	**Beatriz Marín Gimeno**

Friday, August 28

Time	Speaker / Activity
09:00	**Jan Rataj**
10:00	**Eli Putterman**
10:30	Coffee
11:00	**Egor Bakaev**
11:30	**Jun O'Hara**
12:00	**Kateryna Tatarko**

Abstracts

Monday, August 24

Some recent linearly invariant stability results

Károly Böröczky

The recent quantitative version of the Prekopa-Leindler by Figalli-van Hintum-Tiba has inspired an avelange of new stability results in terms of volume difference: for example, for the Brascamp-Lieb Variance inequality by Machado-Ramos, for the Minkowski problem by Boroczky-Machado-Ramos, and for the Santalo inequality or the affine isoperimetric inequality in the o-symmetric case by Boroczky-Patsalos-Saroglou. The talk will review some of the related ideas.

Polarization problems and Coxeter systems

Oscar Ortega

In this talk, I will present a proof of the strong polarization conjecture of Ball and Frenkel, posed about 20 years ago. I will also explain how Coxeter systems give rise to unexpected new extremizers. As a by-product, we solve the real polarization problem, which had remained open for more than three decades, and completely characterize its extremal cases. This is joint work with Ángel D. Martínez.

Isotropic decompositions via inverse eigenvectors

Gergely Ambrus

We develop a residue-theoretic framework for studying inverse eigenvectors of a real or complex $n \times n$ matrix with diagonal entries 1, defined by the nonlinear equation

$$M\alpha = \alpha^{-1},$$

where the inverse is taken coordinatewise. The set of solutions $\alpha \in \mathbb{C}^n$ is denoted by $\mathcal{I}(M)$. Our main result is an inverse analogue of the spectral theorem: under natural transversality and properness assumptions on M, the identity operator admits an explicit decomposition into rank-one tensors associated with the inverse eigenvectors:

$$\sum_{\mathcal{I}(M)} c(\alpha)\, \alpha \otimes \alpha = I_n.$$

The coefficients $c(\alpha)$ are explicitly computable and sum to 1. The proof is based on residues of rational differential forms in several complex variables and is inspired by the recent breakthrough solution of the polarization problem by Martínez and Ortega-Moreno. For real correlation matrices M whose off-diagonal entries have modulus strictly less than one, we remove the properness assumption and prove that $c(\alpha) > 0$ for every $\alpha \in \mathcal{I}(M)$. Consequently, the measure

$$\mu_M = \sum_{\mathcal{I}(M)} c(\alpha)\delta_\alpha$$

is a centered discrete isotropic probability measure on $\mathbb{R}^n$, and the inverse eigenvectors form a weighted tight frame. We extend the construction to arbitrary real Gram matrices, diagonal inverse eigenvectors, and weighted inverse-eigenvector equations. Simple trace and arithmetic–geometric mean arguments yield inverse eigenvectors with controlled Euclidean norm and coordinate product. These estimates lead to short proofs of all currently known versions of the real polarization inequalities, together with weighted and matrix-valued generalizations and further geometric and analytic applications.

Log-concavity principles for sections and projections of convex bodies

Leo Brauner

The classical Brunn-Minkowski inequality, essentially stating that the volume is log-concave, has been extended by Lutwak to intrisic volumes of convex projection bodies. We establish a dual geometric inequality for symmetric mean section bodies (a construction introduced by Goodey and Weil). We deduce our result from an Alexandrov-Fenchel type inequality involving a formal difference of convex bodies. Joint work with Oscar Ortega-Moreno and Franz Schuster.

Minimal area circmuscribed quadrangles around convex disks

Ferenc Fodor

One of the oldest problems in convex and discrete geometry is the approximation of plane convex shapes by circumscribed polygons of minimal area. In the talk, we consider minimal area quadrilaterals circumscribed about convex disks K. It has been known for a long time (Chakerian and Lange, 1971) that the area of such a minimal quarilateral is at most $\sqrt{2}\,\mathrm{area}(K)$. However, this is not optimal as W. Kuperberg (1983) showed that the minimum is strictly less than $\sqrt{2}\,\mathrm{area}(K)$, but he did not provide an explicit improvement.

We show, using a stability argument, that for every convex disk K, there exists a quadrilateral circumscribed about K whose area is less than $(1-2.6\cdot 10^{-7})\sqrt{2}\,\mathrm{area}(K)$. With this, we (slightly) improve the result of W. Kuperberg (1983).

The question of the exact minimum remains wide open and, despite its seemingly simple nature, appears to be notoriously difficult. With this talk we intend to draw attention to this venerable old problem.

This is joint work with Florian Grundbacher (TU Munich).

Bridges between convex geometry and elliptic PDE's

Andrea Colesanti

The overall goal of the talk is to describe some striking analogies between classical results in convex geometry, and corresponding facts in the field of elliptic PDE's. I will start by recalling the Brunn-Minkowski inequality; the variational formula for the volume and the Minkowski problem, in convex geometry. Then I will describe the corresponding results in the PDE's area, when the volume is replaced by well known functionals of the Calculus of Variations, which can be defined through an elliptic boundary value problem. In the final part of the talk I will speak about the corresponding situation in the Gauss space, mentioning some recent results, and open problems.

Brunn-Minkowski type inequalities for the covering radius

Ansgar Freyer

The covering radius of a convex body K is the minimum dilation factor μ such that the translations of $\mu * K$ by a fixed lattice cover the space. It is a very good parameter to measure the size of a convex body relative to a lattice and, thus, oftentimes serves as a bridge between continuous and discrete problems in convexity.

A question that frequently arises in this context is how the covering radius interacts with Minkowski sums of convex bodies. Since the covering radius is a (-1)-homogeneous functional, one might wonder if an inequality analogous to the Brunn-Minkowski inequality holds true. Unfortunately, this is not the case without further assumptions on the relative positions of the bodies in question. In the planar case, we will show that the natural Brunn-Minkowski inequality holds true up to unimodular transformations. Moreover, we present a tight inequality for the covering radius of the Minkowski sum of general planar bodies.

In order to obtain results in general dimension, we will introduce minimal covering bodies as a generalization of translative tiles. These are convex bodies that cover the space by integer translations, but do not properly contain another convex body with the same property. We show that these bodies are polytopes and present a lower bound theorem on their facet number. Finally, if time permits, we discuss their relation to other extremal shapes in discrete convexity such as lattice reduced or complete bodies. The talk is based on joint work with Giulia Codenotti and Katarina Krivokuća.

Infinitesimal forms of the Prékopa–Leindler inequality and applications

Jacopo Ulivelli

After presenting (functional) Wulff shapes, we show a new infinitesimal form of the Prékopa–Leindler inequality encoding the Poincaré inequality on the boundary of convex sets and the Brascamp–Lieb variance inequality. The results thus obtained lead to a new approach towards the dimensional Brunn–Minkowski conjecture for log-concave measures, which we show to hold up to a constant, providing a new proof of a result by Kolesnikov and Livshyts. The talk is based on joint work with Sotiris Armeniakos (TU Wien).

Mahler's conjecture and floating bodies

Agam Guberman

A well-known conjecture by Mahler (1938) asks about the convex bodies that attain a minimal volume product. Floating bodies are the subset of a convex body of constant density that remains above the surface of water regardless of the body's rotation. We will speak of the history of Mahler's conjecture, including very recent breakthroughs, and finally, a new result about floating bodies that connects to the conjecture.

A characterization for measure-valued valuations on star bodies

Jorge Santiago Ibáñez Marcos

Valuations with values in measures arise naturally in convex and dual Brunn–Minkowski theory. In this talk, based on *Jorge S. Ibáñez-Marcos, Monika Ludwig, Pedro Tradacete, Ignacio Villanueva (2026)* ***Measure-valued valuations on star bodies****, arXiv:2606.30538*, we present a characterization of non-negative measure-valued valuations on star bodies which are norm-to-weak* continuous. We further characterize geometric properties of the valuation such as being rotation equivariant, homogeneous and locally determined. As a consequence, we obtain a characterization of q-th dual area measure as the only norm-to-weak* valuations which are rotation equivariant, q-homogeneous and locally determined.

Tuesday, August 25

Localization of valuations and Alesker's irreducibility theorem

Jonas Knoerr

Much of the immense progress in Geometric Valuation Theory over the last 30 years is based on Alesker's Irreducibility Theorem, which provides a representation theoretic description of the space of continuous and translation invariant valuations on convex bodies. A key outcome of this description was the introduction of the notion of *smooth valuations*, which enjoy very strong regularity properties and admit a variety of integral and differential geometric descriptions.

In this talk, I will present joint work with Georg C. Hofstätter on a different approach to the Irreducibility Theorem. The starting point is the observation that it can be obtained from the representation of smooth valuations in terms of integration with respect to the differential cycle by a direct computation of the action of the Lie algebra of the general linear group on a suitable space of differential forms. This reduces the proof to the problem of establishing such a representation for smooth valuations, which we obtain from a corresponding result for valuations on convex functions using a localization procedure.

Towards Hadwiger's conjecture via Bourgain slicing

Marius Tiba

In 1957, Hadwiger conjectured that every convex body in $\mathbb{R}^d$ can be covered by 2^d translates of its interior. For over 60 years, the best known bound was of the form $O(4^d\sqrt{d}\log d)$, but this was improved by a factor of $e^{\Omega(\sqrt{d})}$ by Huang, Slomka, Tkocz and Vritsiou.

In this note we obtain the first exponential improvement towards Hadwiger's conjecture from the recent resolution of Bourgain's slicing problem by Klartag and Lehec. More precisely, we prove that, for any convex body $K \subset \mathbb{R}^d$, $e^{-\Omega(d)} \cdot 4^d$ translates of $\mathrm{int}(K)$ suffice to cover K.

In addition to the application to Hadwiger's conjecture, our method also has an application to the geometry of numbers. Ehrhart conjectured in 1964 that a convex body in $\mathbb{R}^d$ centred at the origin whose interior contains no lattice point other than the origin has volume at most $(d+1)^d/d!$ (this bound is attained by a simplex). The best-known upper bound for the volume of K is of the form $e^{-\Omega(\sqrt{d})} \cdot 4^d$, obtained by Huang, Slomka, Tkocz and Vritsiou. We obtain the first exponential improvement towards Ehrhart's conjecture, bounding the volume of K by $e^{-\Omega(d)} \cdot 4^d$.

This is joint work with Marcelo Campos, Peter van Hintum and Robert Morris.

New examples of Kähler packages and non-Archimedean mixed volumes

Semyon Alesker

An algebra with the Kähler package is a graded (or bigraded) algebra satisfying Poincaré duality (PD), Hard Lefschetz (HL) property, and Hodge–Riemann (HR) inequalities. The earliest examples arise from the cohomology of compact Kähler manifolds. Other examples were discovered in the theory of valuations on convex sets and in combinatorics. In this talk, we describe another construction of algebras with the Kähler package. While it is inspired by the theory of valuations on convex sets, the field of real numbers is replaced by p-adic fields. We prove PD, (non-mixed) HL, and a special case of the (non-mixed) HR inequalities, leaving the remaining cases as conjectures. Certain concrete elements of these algebras behave like mixed volumes involving ellipsoids in convex geometry: they satisfy analogous identities and, conjecturally, the Alexandrov–Fenchel inequality. The latter is proved in some very special cases.

Illumination bodies in projective geometries

Florian Besau

The illumination body $\mathcal{I}_\delta(K)$ of a convex body K consists of all points whose convex hull with K adds at most volume δ. In Euclidean space, the volume derivative of $\mathcal{I}_\delta(K)$ as $\delta \to 0^+$ recovers the affine surface area of K.

We extend illumination bodies to Riemannian space forms and projective Finsler geometries via a weighted volume approach. We prove a general limit theorem: the derivative of weighted volume of weighted illumination bodies yields a notion of surface area that coincides with the affine/floating surface area in each geometry, unifying these constructions within a single framework. We also discuss convexity properties, that is, illumination bodies are always convex in the hyperbolic plane, but fail to be convex in spherical geometry.

Joint work with Rotem Assouline and Elisabeth M. Werner

$\mathrm{SL}(n)$ contravariant tensor valuations of orders up to n

Jin Li

In this talk, I will present a complete classification of all $\mathrm{SL}(n)$-contravariant, p-order tensor valuations on polytopes in $\mathbb{R}^n$ for $n \geq p$, without imposing any symmetry assumptions on the tensors. In addition to recovering the known symmetric cases, I will unveil new asymmetric counterparts, which are naturally associated with the cross tensor and the Levi-Civita tensor. Additionally, some Minkowski-type relations appear naturally, extending the classical Minkowski relation for surface area measures. This is joint work with Dan Ma.

On Ball's conjectured Santalo type inequality

Christos Saroglou

We prove that if K is a symmetric and isotropic convex body in $\mathbb{R}^n$, then

$$\int_K \langle x, u\rangle^2\, dx \int_{K^\circ} \langle x, u\rangle^2\, dx \leq \left(\int_{B_2^n} \langle x, u\rangle^2\, dx\right)^2, \qquad \forall u \in \mathbb{R}^n,$$

with equality for some $u \neq o$, if and only if K is a Euclidean ball. This confirms a conjecture by Keith Ball (1986), stating that for any symmetric convex body K in $\mathbb{R}^n$, it holds

$$\int_K \int_{K^\circ} \langle x, y\rangle^2\, dx\, dy \leq \int_{B_2^n} \int_{B_2^n} \langle x, y\rangle^2\, dx\, dy,$$

with equality if and only if K is an origin symmetric ellipsoid.

Joint work with K.J. Böröczky and K. Patsalos.

Inversion of the two-data Funk transform.

Rafik Aramyan

It is known that the Funk transform (FT) is invertible in the class of even (symmetric) continuous functions defined on the unit 2-sphere S^2. In this article, for the reconstruction of f from $C^1(S^2)$ (which can be non-even), an additional condition is found, which is a weighted Funk transform (to reconstruct an odd function), and the injectivity of the so-called two data Funk transform is considered. The transform consists of the classical FT and the weighted FT. An iterative inversion formula of the transform is presented. Such inversions have theoretical significance in convexity theory, integral geometry, and spherical tomography.

Wednesday, August 26

On Minkowski's monotonicity problem

Ramon van Handel

I will discuss an old open question in convex geometry that dates back to the work of Minkowski in the early 1900s: what are the equality cases of monotonicity of mixed volumes? The problem is equivalent to that of providing a geometric characterization of the support of mixed area measures. A conjectural characterization was put forward by Schneider (1985), but had previously been verified only for special classes of convex bodies. With Shouda Wang, we obtained the first general results in this direction: we fully resolve the conjecture in three dimensions, and fully resolve one direction of the conjectured characterization in every dimension. I will aim to explain these results, as well as the apparently fundamental difficulty that arises in passing from dimension 3 to 4.

From the B-conjecture to the Faber-Krahn position of convex bodies

Dmitry Faifman

The Kac formula provides a well-trodden path between the gaussian measure and the first Dirichlet eigenvalue of domains in $\mathbb{R}^n$. We will follow this path to establish some new inequalities for the first Dirichlet eigenvalue of convex bodies. In particular, departing from the Gaussian B-conjecture, we arrive at a new convexity property of the first eigenvalue of general convex bodies. As a corollary, we deduce that the so called Faber-Krahn position of a convex body, minimizing the first eigenvalue in its SL-orbit, is unique up to orthogonal transformations, thus answering a question of Schmuckenschläger from 2011. This is a joint work with Iosif Polterovich.

Large signed vector sums, polarization problems, and projection constants of convex bodies

Florian Grundbacher

Vector balancing problems have been studied extensively for at least half a century, whereas their counterparts, anti-balancing problems, recieved much less attention. We make up for this deficit by considering the following problem: Given a (Minkowski) norm on $\mathbb{R}^d$ and an integer n, what is the largest constant $C(n)$ such that every set of n unit vectors has a signed sum of norm at least $C(n)$? We establish lower and upper bounds on $C(n)$ for all norms and all d and n, identifying the maximum norm as a sharp minimizer, and the Euclidean norm as an asymptotic maximizer (i.e., up to an absolute constant). Along the way, we show connections to several other concepts in convexity, including polarization problems, intrinsic volumes, containment/approximation problems, and in the limit 1-absolutely summing and projection constants.

This is joint work with Gergely Ambrus (University of Szeged & Rényi Institute).

On the measure of central sections of the n-cube

Bernardo González Merino

The investigation of the volume, surface area, and other geometric properties of sections of convex bodies, and in particular cubes, has a long history and a rich literature. For instance, it is well known that the Lebesgue measure of the central sections of the cube forms an increasing sequence for every $n \geq 3$. However, much less is known when the cube has a volume distribution that is different from the Lebesgue measure; for example, a Gaussian density.

We study the probability densities in the standard cube $B_\infty^n = [-1,1]^n$ of $\mathbb{R}^n$ generated by $e^{-b\|x\|^2}$, $b > 0$. We prove that the limit of the induced Gaussian-type volume of hyperplane sections of B_∞^n through the origin and orthogonal to a main diagonal is

$$\sqrt{\frac{b}{\pi}}\left(1 - 4\frac{e^{-b}\sqrt{b}}{2\sqrt{\pi}\mathrm{erf}(\sqrt{b})}\right)^{-\frac{1}{2}},$$

as $n \to \infty$.

This extends the well-known result of Hensley (1979) for the Lebesgue measure and continues the investigations initiated by Barthe, Guédon, Mendelson, Naor (2005), Zvavitch (2008), and König, Koldobski (2013).

This is a joint work with Ferenc Fodor.

The volume of zonoids: submodularity, Bézout theorem and beyond

Katherina von Dichter

We prove a log-submodularity-type inequality for zonoids in $\mathbb{R}^4$, extending the three-dimensional result of Fradelizi, Madiman, Meyer, and Zvavitch. The inequality is equivalent to a Bézout-type inequality for mixed volumes, a local Loomis–Whitney-type inequality, and an Aleksandrov–Fenchel type estimate, thereby unifying several geometric perspectives. The proof reduces the problem to a combinatorial inequality on the vertices of a cube, establishing unexpected connections between mixed volumes of zonoids, matroid theory, and real algebraic geometry.

Thursday, August 27

Sharp Stability of the Prékopa-Leindler inequality

Peter van Hintum

The Prékopa-Leindler (PL) inequality serves as a functional analogue of the Brunn-Minkowski (BM) inequality on the size of sumsets in $\mathbb{R}^n$. Since PL is obtained from BM in the limit as the number of dimensions tends to infinity, it yields dimensionally independent results in geometry, analysis, and probability, motivating the considerable attention its stability has garnered in recent years. While substantial progress has been made in the geometric setting of BM, a sharp quantitative stability result for the Prékopa-Leindler inequality has remained out of reach. In this talk, we will introduce these inequalities and present recent results that resolve this long-standing open question. Based on joint work with Alessio Figalli and Marius Tiba.

Shadow systems, decomposability and isotropic constants

Christian Kipp

We discuss how necessary conditions for local maximizers of the isotropic constant can be derived from a generic dimension-counting argument if the underlying family of perturbations is parameterized by a vector space. We give a new proof of a decomposability criterion by Campi-Colesanti-Gronchi (in terms of RS-movements) and discuss how this criterion can be extended to a larger class of shadow systems. Using the same proof strategy, we show that the polar body of a local maximizer cannot have "too many" Minkowski summands; more precisely, its dimension of decomposability is at most $\frac{1}{2}(n^2+3n)$. In the polytopal case, the optimality criteria turn out to be affine rigidity conditions.

Longest arcs with the increasing chords property

Zsolt Langi

We say that a curve γ satisfies the increasing chords property, if for any points a, b, c, d in this order on γ, the distance of a, d is not smaller than the distance of b, c. Binmore asked the question in 1971 if there is a universal constant C such that for any curve γ in the Euclidean plane, satisfying the increasing chords property, if the distance of the endpoints of γ is one, then the arclength of γ is at most C. Rote proved in 1991 that the optimal such constant is equal to $\frac{2\pi}{3}$. In this note we give an estimate for the arclengths of curves with the increasing chords property in Euclidean d-space, and generalize Rote's result for such curves in a normed plane with a strictly convex norm. Joint work with Adrian Dumitrescu and Sara Lengyel.

Godbersen conjecture and L_p Rogers–Shephard inequality

Mohamed Mouamine

The classical Rogers–Shephard inequality gives a sharp upper bound for the volume of the difference body $K-K$ in terms of the volume of K, with equality exactly for simplices. Expanding the left-hand side into mixed volumes, Godbersen conjectured in 1938 a termwise refinement of this bound, again extremized by simplices. In this talk, I will present joint work with Jan Kotrbatý proving Godbersen's conjectured inequality for arbitrary convex bodies. The proof is based on mixed-volume monotonicity in a product space. For convex polytopes, analyzing the equality case of this monotonicity yields restrictions on the facial structure of the polytope that ultimately force it to be a simplex, establishing the conjectured characterization of the extremizers among convex polytopes. Finally, I will explain how Godbersen's inequality leads to the sharp L_p Rogers–Shephard inequality, together with the characterization of its equality cases. Joint work with Jan Kotrbatý.

The VC dimension of partial concept classes via Radon's theorem

Marton Naszodi

Following *Alon, Ben-David, Cesa-Bianchi,* and *Haussler*, we define a *partial concept class* (PCC) as a family of partial functions $V \to \{0, 1, *\}$; its Vapnik–Chervonenkis (VC) dimension is defined through shattering in which the undefined value $*$ is forbidden on the shattered set. We study two geometric PCCs in a (possibly infinite-dimensional) real Banach space X, each carrying a margin $\delta > 0$: the *expanded half-spaces*, and the *expanded balls* of unit radius with centers in a fixed ball. We prove bounds on their VC dimension that are *dimension-free*: they depend on the margin δ (and, for balls, on the radii), but not on $\dim X$.

Our main contribution concerns the PCC of expanded balls in ℓ_p and $L_p(\mu)$ with $1 \le p < \infty$, which extends the work of *Bourneuf, Charbit,* and *Thomassé* who studied the PCC of expanded balls in Euclidean space, that is, ℓ_2^d. As an application we derive a Dense Neighborhood Lemma in these spaces.

Joint work with Grigory Ivanov and Attila Jung.

The Mahler conjecture in three dimensions

Dongmeng Xi

The Mahler conjecture dates back to 1938. We recently solved the conjecture for general convex bodies in three dimensions by developing a method called the shadow flow. The equality case is also characterized. This method further gives a new proof of the three-dimensional symmetric case, which was first proved by Iriyeh and Shibata. This talk is based on joint work with Shibing Chen, Yuanyuan Li, and Zhe-Feng Xu.

Weighted and fractional covering numbers

Alon Schejter

For two compact bodies K and T in $\mathbb{R}^n$, the classical *covering number* $N(K,T)$ is the minimal number of translates of T needed to cover K. Extending this concept, the *weighted covering number* $N^w(K,T)$ is the infimal total weight of a *weighted covering*, where translates of T are assigned weights such that each $x \in K$ is covered by translates with a combined weight of at least 1. Similarly, the *fractional covering number* $N^*(K,T)$ is defined as the infimal total mass of a *fractional covering*: a non-negative Borel measure on $\mathbb{R}^n$ such that the measure of the set $\{y \in \mathbb{R}^n : x \in y+T\}$ is at least 1 for every $x \in K$. While many results concerning these parameters are well-established, we show that $N^w(K,T) = N^*(K,T)$ by utilizing linear programming duality within the relatively new framework of *functional covering*. This bridges a gap in the literature, establishing a duality with separation numbers alongside several consequential results, including the limit $N^w(K,T) = \lim_{n\to\infty} N(K^n, T^n)^{1/n}$.

The local logarithmic Brunn-Minkowski inequality for bodies of revolution

Luca Iffland

The conjectured logarithmic Brunn-Minkowski inequality is a stronger version of the classical Brunn-Minkowski inequality, that has many applications in convex geometry, stochastis and other fields of mathematics. In my recent work, I prove a local version of the logarithmic Brunn-Minkowski inequality for one origin symmetric body of revolution and one body that does not need to be symmetric. The proof uses an operator theoretic approach together with the decomposition of spherical functions into isotypical components with respect to rotations around a fixed axis.

On isoperimetric local Bollobás and Thomason inequalities

Beatriz Marín Gimeno

Bollobás and Thomason proved a type of isoperimetric problem relating the volume of the projections of a convex body (a convex and compact set with non-empty interior in $\mathbb{R}^n$) onto coordinate subspaces and the volume of the projections of an appropriate coordinate box, which depends on K, onto the same subspaces.

In this talk we show the next isoperimetric type result which extends the one proved by Bollobás and Thomason. Given $K \subseteq \mathbb{R}^n$ a convex body and $\sigma \subset [n] := \{1, \ldots, n\}$, there exists a suitable Hanner polytope B_K, which depends on K, for which

1. $\mathrm{vol}_n(K) = \mathrm{vol}_n(B_K)$,
2. $\mathrm{vol}_{n-|\sigma|+|\tau|}(P_{H_\sigma^\perp \oplus H_\tau} K) \geq \mathrm{vol}_{n-|\sigma|+|\tau|}(P_{H_\sigma^\perp \oplus H_\tau} B_K)$ for every $\tau \subseteq \sigma$, and
3. $\mathrm{vol}_{n-|\sigma|}(P_{H_\sigma^\perp} K) = \mathrm{vol}_{n-|\sigma|}(P_{H_\sigma^\perp} B_K)$.

Above $P_{H_\tau} K$ means the orthogonal projection of K onto the subspace $H_\tau := \langle e_i : i \in \tau \rangle$ for any $\tau \subseteq [n]$, with $\{e_i\}_{i=1}^n$ being the canonical basis of $\mathbb{R}^n$, and $\mathrm{vol}_n(\cdot)$ means the n-dimensional volume or the Lebesgue measure.

This is a joint work with Luis J. Alías and Bernardo González Merino.

Friday, August 28

On two-dimensional sets with positive reach in $\mathbb{R}^n$

Jan Rataj

Sets with positive reach (PR sets) introduced by Federer in 1959 represent a common extension of the classes of convex bodies and compact C^2 domains. They are currently of interest in different areas of mathematics as integral geometry, Riemannian geometry, metric geometry or data analysis.

In contrast to convex bodies or C^2 domains, PR sets lack an easy characterization of their local structure, even a topological. We will give an overview of known partial result in this direction, as well as a current contribution obtained jointly with Luděk Zajíček - a local characterization of two-dimensional PR sets in $\mathbb{R}^n$. Such sets can be obtained from planar PR sets containing a fixed segment, applying (independent) rotations around this segment to its components separated by one-dimensional points, and a common $C^{1,1}$ diffeomorphism.

Dimension-free stability estimates for the (B)-theorem

Eli Putterman

The (B)-theorem of Cordero-Erausquin, Fradelizi and Maurey states that if γ_n denotes the standard Gaussian in dimension n, K is an origin-symmetric convex set, one has for any t that $\gamma_n(e^t K)^2 \geq \gamma_n(K)\gamma_n(e^{2t}K)$. Herscovici, Livshyts, Rotem and Volberg proved a stability version of this result, showing that if one has equality up to a factor $(1+\delta)$ then the inradius of K must be either "very large" or "very small", where the bounds depend on δ and on n. We will present a new, sharp, stability estimate which is dimension-free and also yields more precise information about bodies which are near-optimizers: every principal component of the covariance matrix of the measure obtained by restricting the Gaussian to K must either be at least $1-O(\delta)$ or at most $O(\delta)$. Our method extends immediately to yield stability estimates for generalizations of the (B)-theorem, namely the "strong" and "functional" versions, which reduce to spectral questions about 1-log-concave measures on $\mathbb{R}^n$.

An observation on the affine plank conjecture

Egor Bakaev

In 1951, Bang posed the *affine plank conjecture*, which remains open: If a convex body in $\mathbb{R}^d$ is covered by planks, then the total relative width of the planks is at least one. We prove a lower bound of $2/(1+\sqrt{d})$ for this total relative width. The best previously known lower bound was $2/(1+d)$.

Distance invariants and the reconstruction of planar polygons

Jun O'Hara

For a compact body in Euclidean space, several distance-based invariants can be associated with it. Among them are the Riesz energy function, defined by the double integral of a complex power of the distance, the cumulative distribution function of pairwise distances, and, in the convex case, the cumulative distribution function of chord lengths. The equivalence between the latter two is classical, while the relation with the Riesz energy function follows from Mellin transform techniques and analytic continuation.

Motivated by inverse problems in integral geometry, we investigate to what extent these invariants determine the original body. Restricting ourselves to planar polygons, we show that a generic polygon can be reconstructed from either the Riesz energy function or the distance distribution. This result may be regarded as a non-convex analogue of Waksman's theorem on Blaschke's chord-length problem.

The underlying principle is that genericity prevents different geometric data from being merged into the same invariant. This viewpoint also appears in the discrete setting of finite metric spaces, where generic spaces can be reconstructed from spectral invariants.

Spherical Grunbaum's inequality

Kateryna Tatarko

A classical inequality of Grunbaum in $\mathbb{R}^n$ gives a sharp lower bound for the ratio between the volume of the portion of a convex body cut off by a hyperplane passing through its centroid and the volume of the entire convex body.

In this talk, we discuss the analogue of Grunbaum's inequality on the sphere $\mathbb{S}^{n-1}$. The inequality we obtain is optimal. Joint work with S. Myroshnychenko, D. Ryabogin and V. Yaskin.